Analysis of Source Location Accuracy Using a Network of Acoustic Sensors with Application to Network System Design

Dr S Harman, Dr A Hume, Mr V Calloway, Mr D Beale
QinetiQ, St Andrews Rd, Malvern, Worcestershire, WR14 3PS, UK

saharman@QinetiQ.com, vpcalloway@QinetiQ.com

ABSTRACT

Recent advances in computer processing power and network communications have brought acoustic sensing back into the forefront of battlefield surveillance. In particular forward-deployed networks of unattended ground sensors are becoming increasingly popular. Using these networks it is envisaged that vehicle tracking, identification and gun battery location could be achieved. Along with a multitude of new processing options comes a vast number of decisions that will determine how cost efficient and robust the system will be. Design parameters such as sensor density, dimensions of each network node, processing power required at each node, number of sensors on each node all affect the cost and effectiveness of the system. In this paper a method of determining the sensor density required for a given system accuracy is presented. From simulation and statistical analysis of the bearing-calculation and triangulation functions, the accuracy of the source location estimate for any network configuration can be calculated. Using this technique we are able to determine the number of sensors needed to cover an area to the required accuracy. Using the same analysis the array configuration on the node can be determined so that for the given dimensions of the array (which is often restricted by the deployment method) an optimal configuration can be produced. In this paper the statistical methods used for the location accuracy analysis, and the relationship to the sensor density and array configuration are presented.

1.0 INTRODUCTION

Unattended ground sensors (UGS) have undergone a renaissance due to the advances in mobile phone technologies. Advances in communications now make it possible to distribute large numbers of sensor nodes to form ad-hoc networks capable of transmitting sensor data from remote parts of the battlefield back to the central command post. These advances have opened new possibilities in the way that battlefield sensing is approached. In particular there is potential for individual vehicles or battle groups to be tracked through the whole sensor coverage area helping to form the digitisation of the battlefield and thus supporting the Netted Enabled Capability (NEC) initiatives. It is hoped that in the future the NEC will provide increased, persistent and robust coverage of the battlefield. Along with this they must offer the same capability as current in-service sensors systems.

The requirements the network of UGS must fulfil can be split into the sensing requirements and the system requirements. The sensing requirements demand an unambiguous tracking capability along with a weapon locating capability, both of which must be achieved to a required accuracy. In order that the concept is viable the system requirements demand that UGS be easily deployable and hence light and small and that they must also be disposable hence cheap. The acoustic sensor presents itself as a sensor technology capable of fulfilling many of the requirements. In terms of sensing requirements small arrays of acoustic sensors can be used for tracking and they have a well established pedigree in weapon locating systems. In terms of system requirements they are passive, low power, inexpensive, provide non-line-of-sight coverage and are robust. However the system requirements concerning UGS and network infrastructure conflict with demands on the overall system accuracy. The necessity of small nodes for easy

Paper presented at the RTO SET Symposium on "Capabilities of Acoustics in Air-Ground and Maritime Reconnaissance, Target Classification and Identification", held in Lerici, Italy, 26-28 April 2004, and published in RTO-MP-SET-079.

deployment restricts the size of the on-node acoustic array, which is fundamentally linked to tracking accuracy. Also the number of nodes used will effect the cost, which in turn effects the overall accuracy of the UGS network.

The purpose of this paper is to provide a preliminary analysis into the effect of sensor node density on the overall accuracy of the UGS network. Models for assessing acoustic sensor performance in fixed scenarios will be extended to produce performance metrics based only on generic sensor density information. In the case of weapon locating the metric is positional accuracy and for vehicle tracking the metric is the percentage track coverage through the sensor network.

2.0 ACOUSTIC WEAPON LOCATING

There are two weapon locating (WL) techniques that can be applied using acoustic sensors. The first relies on each node in the network to be a small acoustic array capable of generating a bearing to the sound source. In this way bearings from several distributed nodes are triangulated to produce the acoustic source location. The second method requires only a single microphone at each node in the network. The source location in this case is calculated using the assumption that the acoustic energy propagates in a circular pattern and the time at which the acoustic energy passes over each sensor node. Both methods are described in greater detail in the sections that follow.

2.1 WL Using Arrays of Microphones on the Nodes

This method for source location depends on the microphone arrays being small enough, or far enough away from the source, for the wavefront to appear planar. When this is the case individual nodes provide the bearing of the acoustic wavefront. The equations that define the TDOA over the small acoustic array are dependent only on the wavefront bearing:

$$\tau_{m1} = \frac{(x_1 - x_m)\cos(\theta) + (y_1 - y_m)\sin(\theta)}{c} \tag{1}$$

where (x_i, y_i) are the relative positions within the on-node array of the sensors, τ_{m1} is the time difference of arrival (TDOA) between sensors in the on-node array, and θ is the wavefront bearing. Figure 1 shows the geometry of the TDOA equation. This equation solves analytically to give the source bearing.

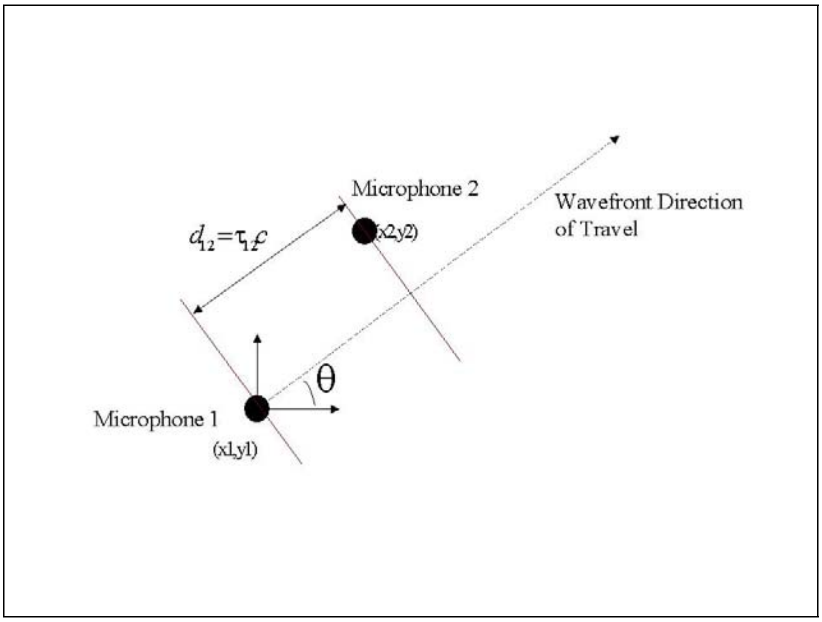

Figure 1 Geometry of the planar wavefront propagating over sensors in the on-node array

When the bearing has been calculated by several nodes in the network, triangulation is used to calculate the source coordinates. The triangulation equations are:

$$X_s = \frac{Y_2 - X_2\tan(\theta_2) - Y_1 + X_1\tan(\theta_1)}{\tan(\theta_1) - \tan(\theta_2)} \tag{2}$$

$$Y_s = \frac{[Y_2 - X_2\tan(\theta_2)]\tan(\theta_1) - [Y_1 - X_1\tan(\theta_1)]\tan(\theta_2)}{\tan(\theta_1) - \tan(\theta_2)} \tag{3}$$

Where (X_i, Y_i) are the coordinates of node *i* and θ_i is the bearing from the i^{th} node to the target.

2.2 WL Using Single Microphone Nodes

This method of source location depends on the acoustic wave travelling in a circular motion. From the time at which the sound reaches each node a set of simultaneous equations are derived that solve to give the coordinates of the gun position. The equations derived from the times of arrival at the nodes are given by:

$$\tau_{i1} = \frac{\sqrt{(X_1 - X_s)^2 + (Y_1 - Y_s)^2} - \sqrt{(X_i - X_s)^2 + (Y_i - Y_s)^2}}{c} \tag{4}$$

Where (X_i, Y_i) is the position of the i^{th} node, τ_{i1} is the measured TDOA between node *i* and node 1 and *c* is the speed of sound. If there are *M* sensor nodes there will be *M-1* simultaneous equations that solve to give the source location (X_s, Y_s).

In general, when the number of nodes exceeds three, there will be no closed form solution to this set of simultaneous equations, so an iterative numerical solution must be used. Here we use Taylor's method [1].

3.0 ACOUSTIC VEHICLE TRACKING

Vehicle tracking is achieved by using an array of sensors at each node to generate a sequence of bearings over time. The type of array used is very similar to that used in the first WL technique discussed in section 0. However, in this case, the acoustic signature is not impulsive (as is the case for the gun break) but continuous. Here impulse detection techniques for measuring the TDOA must give way to more sophisticated continuous signature processing. These techniques depend on each sensor in the array recording time shifted data of the same acoustic signature. Again the time shifts are defined by equation (1). Beamforming techniques are one way of manipulating the time shifted array recordings to estimate the bearing of the acoustic wavefront. Examples of these algorithms include MUltiple SIgnal Classification (MUSIC), Fourier, Multi-Variate Distortionless Response (MVDR), and the Maximum Entropy Method (MEM). Details of these techniques are can be found in [2]. The preferred method currently being used by the authors is the MUSIC algorithm.

The sequence of bearings recorded at multiple nodes triangulate to give the instantaneous source coordinates. The triangulation is precisely the same as in the WL equations (2) and (3). Over time a track of the vehicle motion emerges, as illustrated in Figure 2.

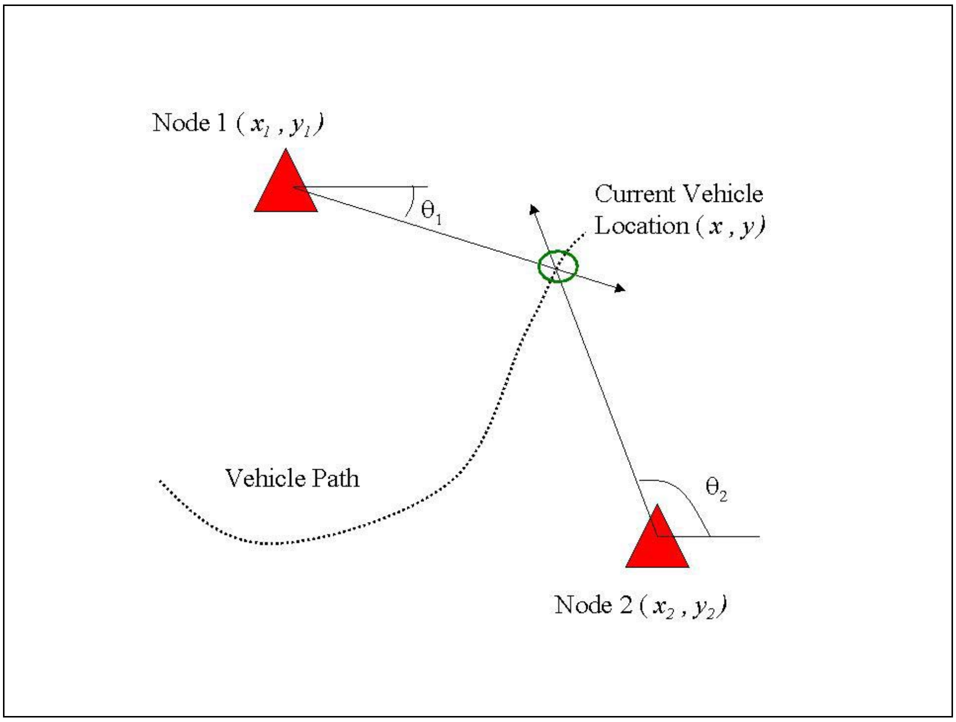

Figure 2 Triangulation using node location and bearing data

4.0 SENSOR NETWORK PERFORMANCE

4.1 Fixed Configuration for Weapon Locating

From the equations outlined in sections 0 and 0 we see that the coordinates of the target are calculated from several input parameters. These parameters include sensor positions, node positions, the speed of sound and the TDOA. In practice each of these parameters will incur measurement errors. For the node positions the error may come form GPS inaccuracies, the weather effects the speed of sound and the TDOA has measurement errors related to the signal to noise ratio. All these errors propagate through the location calculations to cause errors in the derived target coordinates. The extent of this error can be calculated using laws on the propagation of errors found in random variable theory [3].

When the target location can be expressed analytically, as in the above equations, the location error is calculated from the errors of the input variables. If the location coordinates are given by:

$$x = f(p_1, p_2, \ldots, p_N) \tag{5}$$

$$y = g(p_1, p_2, \ldots, p_N) \tag{6}$$

where p's are the measured input parameters, then the derived location errors, represented as the position covariance matrix, are given by:

$$\sigma_x^{\ 2} = \sigma_{p_1}^{\ 2}\left(\frac{\partial x}{\partial p_1}\right)^2 + \sigma_{p_2}^{\ 2}\left(\frac{\partial x}{\partial p_2}\right)^2 + \ldots + \sigma_{p_N}^{\ 2}\left(\frac{\partial x}{\partial p_N}\right)^2 \tag{7}$$

$$\sigma_y^{\ 2} = \sigma_{p_1}^{\ 2}\left(\frac{\partial y}{\partial p_1}\right)^2 + \sigma_{p_2}^{\ 2}\left(\frac{\partial y}{\partial p_2}\right)^2 + \ldots + \sigma_{p_N}^{\ 2}\left(\frac{\partial y}{\partial p_N}\right)^2 \tag{8}$$

$$\sigma_{xy}^{\ 2} = \sigma_{p_1}^{\ 2}\frac{\partial x}{\partial p_1}\frac{\partial y}{\partial p_1} + \sigma_{p_2}^{\ 2}\frac{\partial x}{\partial p_2}\frac{\partial y}{\partial p_2} + \ldots + \sigma_{p_N}^{\ 2}\frac{\partial x}{\partial p_N}\frac{\partial y}{\partial p_N} \tag{9}$$

where σ_x is the standard deviation of the derived x position, σ_y is the standard deviation of the derived y position, $\sigma_{xy}^{\ 2}$ is the covariance of the derived x and y position and σ_{p_i} is standard deviation of the i^{th} input parameter.

Equations (7) to (9) define the error in statistical terms, however the standard value quoted for derived accuracy is the Circular Error Probability (CEP). The CEP summarises the statistical error as a single parameter radius. The higher the CEP the greater the derived location error, and the poorer the system performance. The method for calculating CEP from covariance is outlined in the Appendix. An example of a WL performance, using a sensor deployment tool developed by the authors, is shown in Figure 3. Here the sensor network represents a typical configuration of an acoustic sound ranging sensors system.

Here the CEP is calculated from known sensor node positions and input the parameter uncertainties. Given the node positions and the target location we can derive the system performance in the form of a CEP. However assessment of a specifically configured sensor network only provides information about that configuration. Instead, general performance of a network for a given sensor *density* is needed. This relates more closely to real applications where the sensor density is known but, due to the nature of the remote deployment, the specific position of the sensor nodes cannot be pre-determined.

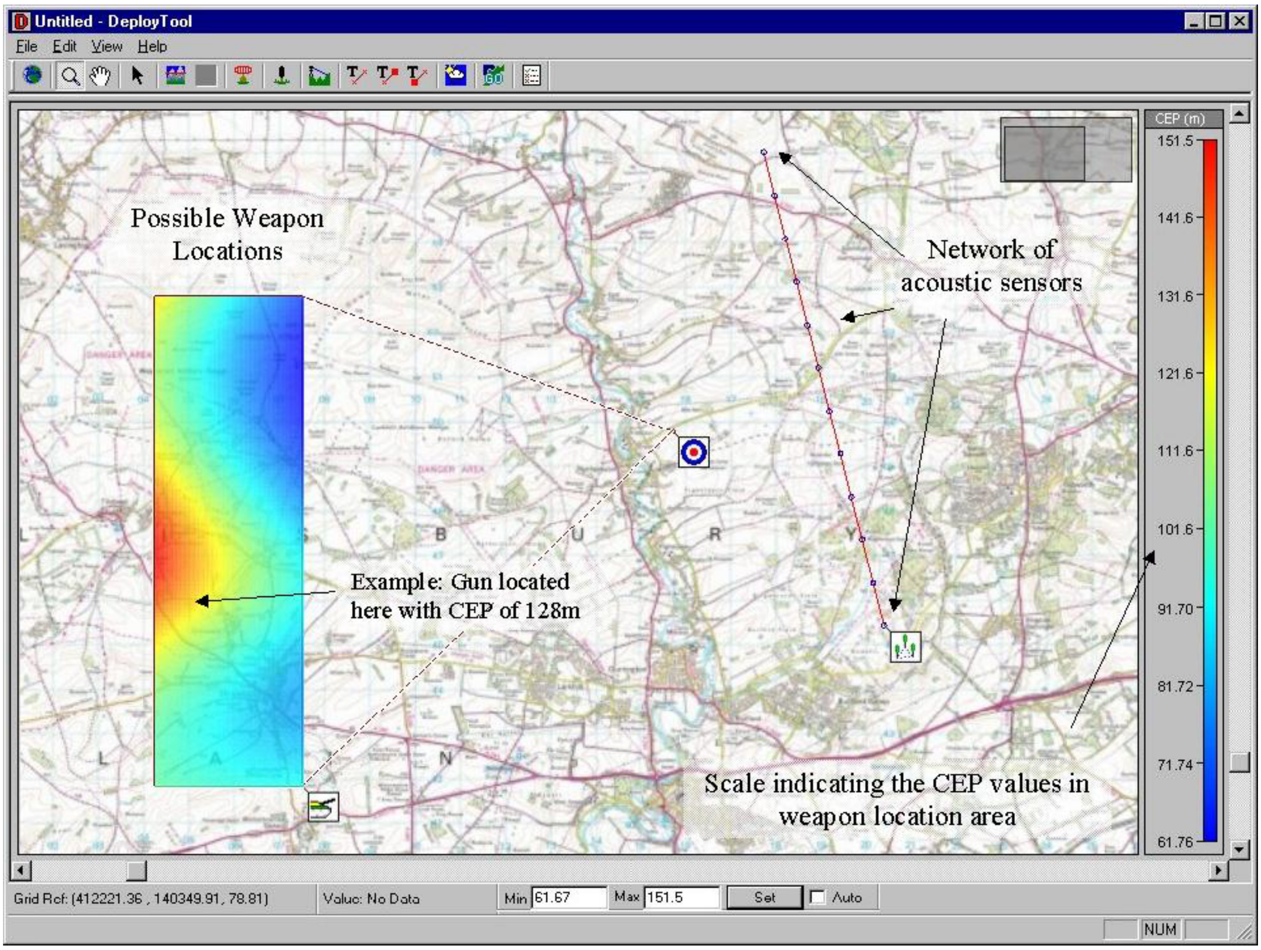

Figure 3 Example of weapon locating accuracy for a fixed sensor network scenario.

4.2 Fixed Configuration for Vehicle Tracking

Assessing the performance of vehicle tracking through an UGS network is more complex than that of WL. For vehicle tracking the target position is constantly changing. Therefore location accuracy, calculated using techniques outlined in 0, will vary with time. This variation will indicate the network detection is more accurate at some points along the vehicle path than at others. Averaging this information does provide a good metric for measuring the tracking performance. If the vehicle is perfectly tracked for some sections of the vehicle path but poorly in others the overall performance may look favourable. However a required capability is that the network maintains a consistent track of the vehicle. Therefore the percentage of the vehicle path competently covered by the sensor network is a more useful metric for determining the tracking performance.

The percentage coverage depends on the effective range of the acoustic sensors. The percentage of the path lying within range of one sensor will indicate detection coverage and when within three sensors will indicate robust tracking coverage. Figure 4 shows how vehicle path coverage may vary through an UGS network. In the figure only a small percentage of the total path lies within range of at least three sensors, but a good proportion lies within detection range of at least one.

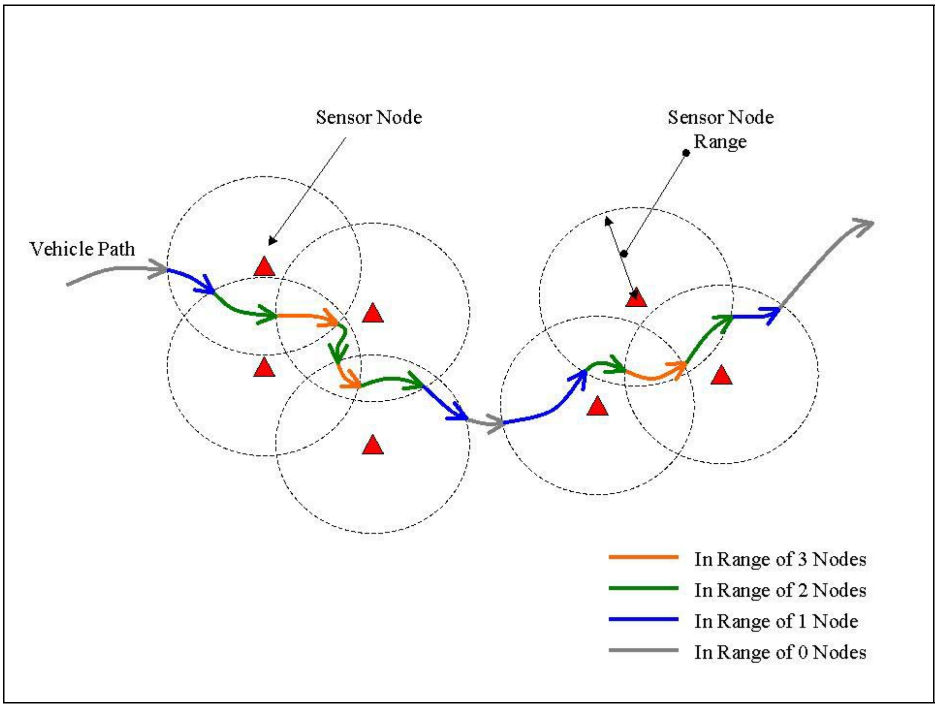

Figure 4 Coverage of vehicle path through an UGS network

As in the case of WL assessment, the percentage path coverage for vehicle tracking is performed for fixed sensor configurations. That is, given a vehicle path and a sensor node configuration the percentage path coverage is calculated. Once again, however, it is the effectiveness of generic sensor densities that is of interest. Therefore for both WL and vehicle tracking the fixed scenario assessment methods must be extended to assess sensor node densities. This can be achieved using the fixed scenario assessment in Monte Carlo simulations.

4.3 Generic Network Performance Assessment

The fixed scenario techniques generate sensor performance metrics, either a CEP or percentage coverage. For the generic case these measures must be derived for a given sensor density where the node positions are unspecified. This is achieved by randomly placing sensor nodes within an area of coverage to the specified sensor density. Using the randomly placed node configuration the system performance can be derived and a CEP or percentage coverage score is generated. The process is repeated several thousand times resulting in a set of performance values. Taking the mean of these values provides the generic sensor performance measure required.

5.0 RESULTS

5.1 Weapon Locating

The results of the Monte Carlo simulations are plotted below.

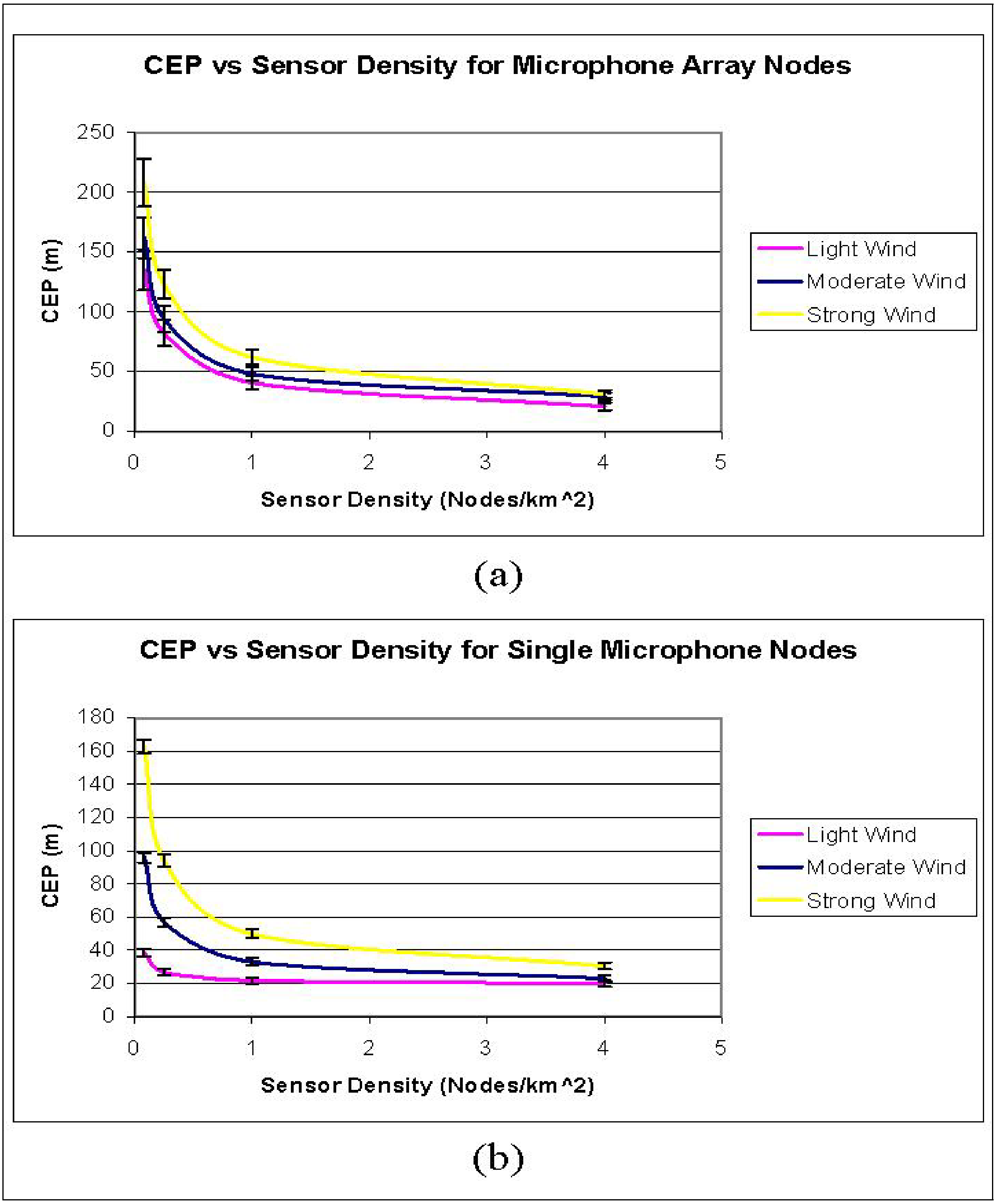

Figure 5 Performance curves for Weapon Locating against sensor density

The results in illustrate how the CEP reduces as sensor density increases and that for varying meteorological conditions the performance also varies. In particular as the wind speed increases the CEP increases. These results indicate system performance for location of an AS90 firing charge 6 with background noise at levels expected in a military environment (90dB).

5.2 Vehicle Tracking

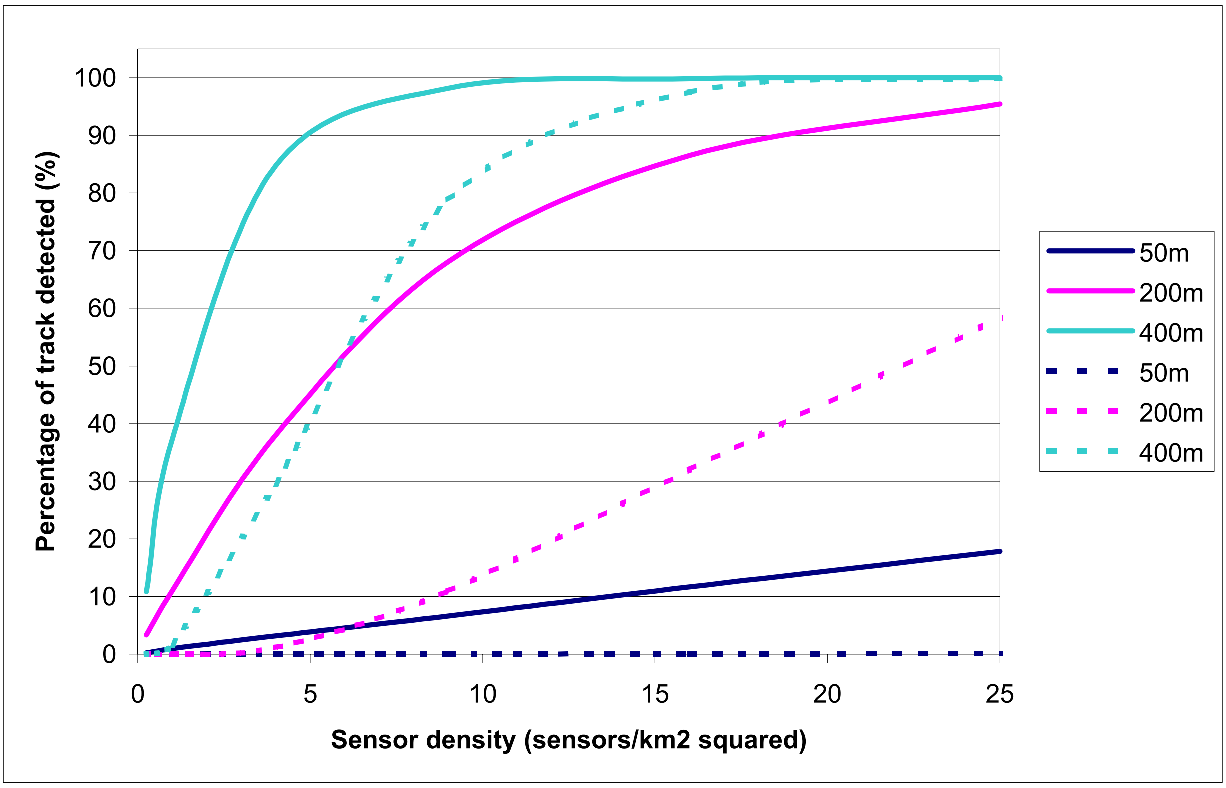

Figure 6 Performance curves for vehicle tracking against sensor density

In Figure 6 the dashed lines represent percentage coverage by at least 3 nodes simultaneously, and the solid lines represent percentage coverage by at least one node. Using this set of curves the density can be determined for a required coverage. For example if a node has an effective tracking range of 400m, and the vehicle is to be within tracking range of at least three nodes simultaneously 80% of the time then 8 nodes per km^2 are required. However if the same percentage coverage is required for a single node at any one time then 2 nodes per km^2 are required.

6.0 CONCLUSIONS

The results of the Monte Carlo simulations clearly show that the effectiveness of UGS networks is directly related to the sensor node densities. So as higher levels of performance are required the sensor density will increase. In addition to this the results show that sensor densities required for robust tracking greatly exceeds the densities required for weapon locating. In consequence it may be concluded that combining the two requirements into a single system may not be advisable.

Paying particular attention to vehicle tracking, the results show that single node coverage of the vehicle can be achieved with a far lower density than that of the 3-node coverage. If bearing only tracking could be used at individual nodes, i.e. tracking in xy Cartesian space using only bearing measurement information, then the demands on sensor numbers can be significantly reduced. However to achieve this the position of the vehicle must be known. Similarly the vehicle tracking performance graphs show that individual node capability is interchangeable with sensor density. In particular it was clear that as the sensor performance range increased from 50 to 400 meters, fewer nodes were required to maintain a comparable level of percentage coverage. In this case improving the sensing range requires increasing the array size, using higher quality components and increasing the numbers of sensors, all of which will increase the cost of the node.

In conclusion we have outlined many of the issues relating to sensor network design and shown how system performance can be related to material costs. In particular we have developed a methodology for assessing the generic sensor system performance based on individual node capability and the network sensor density.

ACKNOWLEDGEMENTS

This paper was produced under funding from MOD ARP 04.

The crown copyright data reproduced here has been done so with the permission of HMSO.

REFERENCES

[1] Aatique M, *Evaluation of TDOA techniques for position location in CDMA systems,* MSc Thesis for Virginia Polytechnic Institute and State University, Chapter 2, Sept 1997. (available at web address: http://scholar.lib.vt.edu/theses/available/etd-82597-03345/unrestricted/aatique.pdf)

[2] Jonson D H and Dudgeon D E, *Array Signal Processing, Concepts and Techniques*, Prentice-Hall1993.

[3] PR Bevington, *Data reduction and error analysis for the physical sciences*, McGraw-Hill New York 1969

APPENDIX

Circular Error Probability (CEP)

The CEP is defined as: the circle radius about the true target location that encompasses 50% of the probability density function. Although obtaining values of the CEP from the covariance matrix is not directly solvable an empirical formula has been derived by the authors:

$$CEP = E_1\left(c_0 + c_1\frac{E_2}{E_1} + c_2\left(\frac{E_2}{E_1}\right)^2 + c_3\left(\frac{E_2}{E_1}\right)^3 + c_4\left(\frac{E_2}{E_1}\right)^4\right) + \varepsilon \quad , \quad E_1 > E_2 \tag{10}$$

where $E_1$and E_2 are the largest and smallest eigen-vectors of the covariance matrix, the parameters c_0 to c_4 are [0.675, -0.13488, 1.7079, -1.6043, 0.53343] and $\varepsilon < 0.01CEP$ when $E_1 < 10E_2$. Figure 7 shows this measure pictorially. The scatter plot in the figure represents all the points generated if the source location algorithms were run many times accounting for the errors in the input variables.

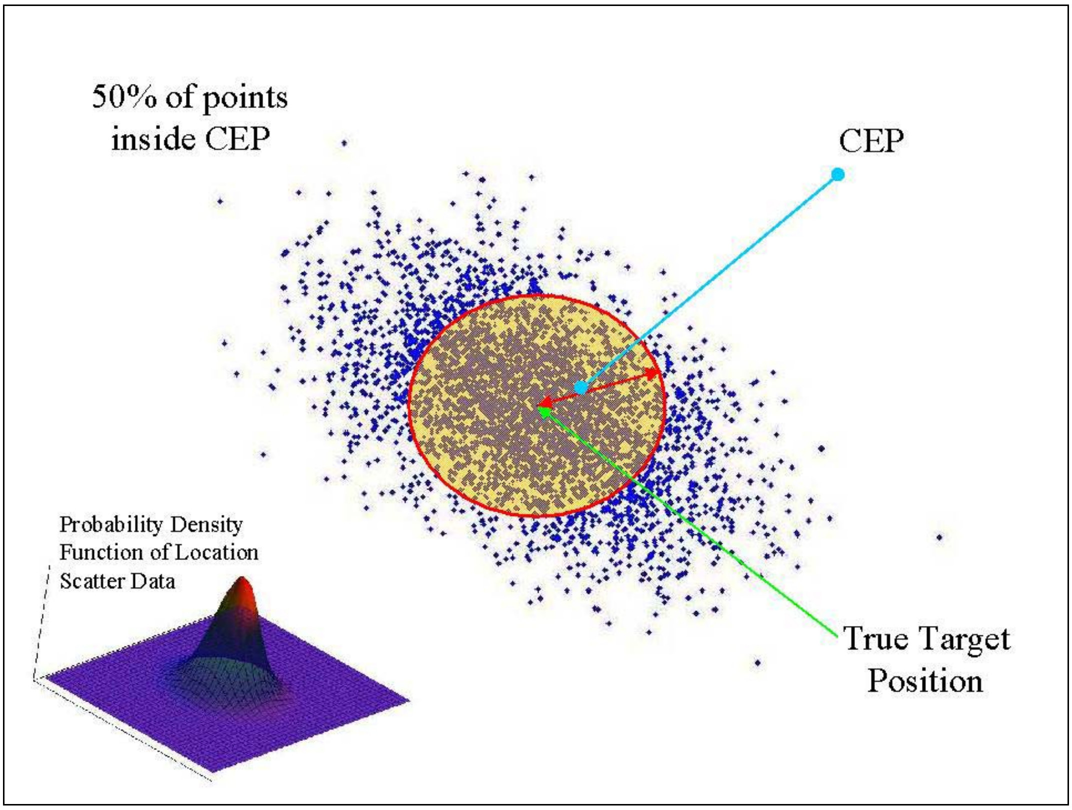

Figure 7 Distribution of the target location error showing the CEP.